

[illegible]

10 | E | I

Second Term Test - Grade 12 - 2020

Index No :

Combined Mathematics I

Three hours only

Instructions:

- * *This question paper consists of two parts.*
Part A (Question 1 - 10) and **Part B** (Question 11 - 17)
- * **Part A**
Answer all questions. Write your answers to each question in the space provided. you may use additional sheets if more space is needed.
- * **Part B**
Answer five questions only. Write your answers on the sheets provided.
- * *At the end of the time allocated, tie the answers of the two parts together so that **Part A** is on top of part B before handing them over to the supervisor.*
- * *You are permitted to remove only **Part B** of the question paper from the Examination Hall.*

For Examiner's Use only

(10) Combined Mathematics I		
Part	Question No	Marks Awarded
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
	Total	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
Paper 1 total		
Percentage		

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Marking Examiner	
Marks Checked by ¹ ₂	
Supervised by	

Combined Mathematics 12 - I (Part - A)

Answer all the questions in Part A and only for five questions in Part B.

- 01) Show that the roots of the equation $(1 - k)x^2 + x + k = 0$ are real and negative if, $0 < k < 1$.

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- 02) Find all real values of x satisfying the inequality $3 - |x + 1| < x^2$

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03) Solve , $3^{2x+1} - 3^{x+4} + 3^3 = 3^x$

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04) Show that , $\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{x^2} = \pi$

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- 05) Solve the simultaneous equations of , $\log_3 x + \log_3 y = 3$ and $\log_y x = 2$

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- 06) Let there is a common linear factor for both polynomials $x^3 + ax^2 + b$ and $ax^3 + bx^2 + x - a$. Show that the above common linear factor is a factor of the polynomial $(b - a^2)x^2 + x - a(1 + b)$ too.

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- 08) Show that $a > \frac{11}{9}$, if both roots of the equation $x^2 - 6ax + 2 - 2a + 9a^2 = 0$ are greater than three.

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Combined Mathematics 12 - I (Part - B)

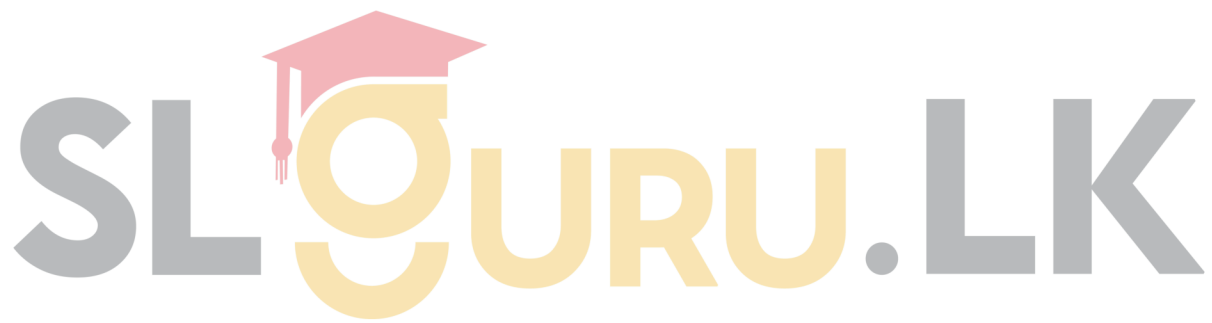
Answer only five questions.

- 11) a) Let $f(x) = x^4 + Px^2 + r$. Find p and r if $f(1) = -9$ and $f(0) = -8$. Find the values of the real constants a, b and c , if $f(x)$ can be expressed in the form $(ax^2 + b)^2 + c$. Here $a > 0$. Hence find the real roots of $f(x) = 0$.
- b) Find the range of values of P , for the expression $(p-1)x^2 - 4x + p-1$ to be positive for all real values of x .
- c) If, α, β are the roots of $ax^2 + bx + c = 0$, find the roots of $cx^2 - 2bx + 4a = 0$ in terms of α, β .
- d) Separate $\frac{x^2-1}{x^2(2x+1)}$ into partial fractions.
- 12) a) Sketch the graphs of $y = 2|x+1| - 3$ and $y = x + 2|x-1|$ in the same diagram. Hence solve the equation $x + 2|x-1| = 2|x+1| - 3$.
Find the set of value of x , satisfying the inequality $x + 2|x-1| > 2|x+1| - 3$.
- b) Let $a = \log_{2n} n$, $b = \log_{3n} 2n$ and $C = \log_{4n} 3n$, for a positive real number n .
Prove that $1 + abc = 2bc$.
- c) If $a^x = b^y = c^z = d^w$, obtain that $x \left(\frac{1}{y} + \frac{1}{z} + \frac{1}{w} \right) = \log_a bcd$.
- 13) a) $f : \mathcal{R} \rightarrow \mathcal{R}$ is defined as follows,

$$f(x) = \begin{cases} -x^4 + 4; & x < 1 \\ -2x; & x \geq 1 \end{cases}$$
 (i) Draw the rough sketch of $f(x)$.
 (ii) Evaluate $\lim_{x \rightarrow 1^-} f(x)$ and $\lim_{x \rightarrow 1^+} f(x)$
 (iii) Is the function continuous at $x = 1$? Justify the answer.
 (iv) Find $\lim_{x \rightarrow 1} f(x)$, if exists.
- b) Prove that $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$
 Evaluate the following limits.
- (i) $\lim_{x \rightarrow 3} \frac{\sqrt{2x-1} - \sqrt{5}}{\sin(x-3)}$
- (ii) $\lim_{x \rightarrow 0} \frac{(\sqrt{4+x^2}-2)(1-\cos 2x)}{x^4}$

- 14) a) Write the conditions which should be satisfied for the roots of the equation $ax^2 + bx + c = 0$ to be real and positive. Here $a, b, c \in \mathcal{R}$ and $a \neq 0$.
When those conditions are satisfied show also that the roots of the equation $a^2x^2 + a(3b - 2c)x + (2b - c)(b - c) + ac = 0$ are real and positive. If the roots of the second quadratic equation are α and β , using a suitable linear transformation, obtain the quadratic equation with roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.
- b) (i) Find k , if the distance between the two points $(k, 2)$ and $(3, 4)$ is 8 units.
(ii) Find the coordinates of the remaining vertex, if $(1, 0)$, $(6, 1)$ and $(5, 6)$ are the vertices of a square separately.
(iii) In the triangle ABC , let $A = (1, 3)$ and $B = (5, 3)$.
Find the coordinates of C , if the coordinates of the centroid of the triangle ABC is $(\frac{10}{3}, 4)$.
- 15) a) State and prove the factor theorem.
If the polynomial $f(x) = x^4 + ax^3 + bx + c$ is divisible by $(x - 1)(x + 1)(x - 2)$, find a, b , and c and find the remaining factors.
Also find the solutions of $2f(x + 1) = x^2 + x - 2$.
- b) Separate the rational function $\frac{x^2}{(x-a)(x-b)}$ into partial fractions in terms of a and b . Hence obtain the partial fractions of $\frac{4x^2}{4x^2 - 1}$.
- c) Evaluate $\lim_{x \rightarrow \infty} (\sqrt{x^2 + ax + a^2} - \sqrt{x^2 + a^2})$
- 16) a) Using the expression for $\sin(A + B)$, show that $\sin(\frac{5\pi}{12}) = \frac{\sqrt{6} + \sqrt{2}}{4}$.
Hence deduce that, $\cos(\frac{5\pi}{6}) = -\frac{\sqrt{3}}{2}$.
- b) If $\alpha + \beta - \gamma = \pi$, Prove that $\sin^2 \alpha + \sin^2 \beta - \sin^2 \gamma = 2 \sin \alpha \sin \beta \cos \gamma$.
- c) Find the general solutions of the equation $2 \cos^2 x + \sqrt{3} \sin x + 1 = 0$.
- d) Obtain that $\tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{1}{3}) = \frac{\pi}{4}$
- 17) State the sine rule for a triangle ABC .
- a) Prove that, $\frac{a^2 + b^2}{a^2 + c^2} = \frac{1 + \cos(A - B) \cos C}{1 + \cos(A - C) \cos B}$
- b) In the triangle ABC , the mid point of the side BC is D . According to the standard notation, show that $AD = \frac{\sqrt{2b^2 + 2c^2 - a^2}}{2}$,
If $\widehat{BAD} = \beta$, show that $\sin \beta = \frac{a \sin B}{\sqrt{2b^2 + 2c^2 - a^2}}$
If $\widehat{ADC} = \theta$, show that $\sin \theta = \frac{2b \sin C}{\sqrt{2b^2 + 2c^2 - a^2}}$

“Education is the foundation upon which we build our future.” —
Christine Gregoire



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10 | E | II

Second Term Test - Grade 12 - 2020

Index No :

Combined Mathematics II

Three hours only

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A	1	
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Percentage		

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Paper II	
Total	
Final Marks	

Final Marks

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In Words	

Marking Examiner	
Marks Checked by ¹ ₂	
Supervised by	

(Part - A)

- 1) The resultant of two forces of magnitude P and $2P$ is $\sqrt{3}P$. Find the angle between the two forces. Also find the angle between the resultant force and the first force.

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- 2) A particle of mass 5kg is on a fixed smooth plane of inclination 30° to the horizontal. Find the magnitude of the force which should be applied parallel to the inclined plane and find the reaction between the inclined plane and the particle.

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- 3) The position vectors of the points A, B and C relative to a fixed point O are $\underline{a}, \underline{b}$ and $\underline{c}$ respectively. If $3\underline{a} + 5\underline{b} = 8\underline{c}$, show that the points A, B and C are collinear. Find the ratio $AC : CB$.

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- 4) A particle of weight 6N is suspended by two light strings and it is in equilibrium. If the tensions of the strings are 3N and $3\sqrt{3}\text{N}$, find the angles which the two strings make with the vertical.

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- 5) A particle P is projected vertically upwards with a velocity $2u$ from a point O in a plane. At the same instant from the point O , a particle Q is projected vertically downwards with velocity u . Both particles move under gravity. Draw the velocity time graphs for the motion of both particles P and Q in the same diagram and show that the velocity of the particle Q is $3u$. When the particle P reaches its maximum height.

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- 6) A particle is projected vertically upwards from a point O , on the ground floor. If the time taken for the particle to travel upwards and downwards passing a point, which is at a vertical height h , is t_1 and t_2 respectively. Show that $h = \frac{1}{2}gt_1t_2$.

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7) $\underline{a}$ and $\underline{b}$ are two unit vectors and the angle between the two vectors is θ . Show that $\sin \frac{\theta}{2} = \frac{1}{2} |\underline{a} - \underline{b}|$.

8) The two ends of a non-uniform rod of mass 4 kg and length 24 m are A and B . The rod is kept in equilibrium horizontally on two pegs which lie at a distance 8 m . The distance to the nearest peg from A is 4 m . When a weight of 6 N is hung at B , the reactions on the rod by the pegs are equal. Find the reaction on the rods. Also find the distance to the center of gravity of the rod from A .

8) The two ends of a non-uniform rod of mass 4 kg and length 24 m are A and B . The rod is kept in equilibrium horizontally on two pegs which lie at a distance 8 m . The distance to the nearest peg from A is 4 m . When a weight of 6 N is hung at B , the reactions on the rod by the pegs are equal. Find the reaction on the rods. Also find the distance to the center of gravity of the rod from A .

- 10) Two motor cars move with uniform velocities v and u towards each other in a straight line path. When the distance between the motor cars is d , both cars apply brakes at the same time knowing that the two cars get collide together. Two cars obtained retardations of f_1 and f_2 respectively, as a result of applying brakes and just prevented the collision. Draw the velocity time graph for the motion. Show that $d = \frac{u^2}{2f_2} + \frac{v^2}{2f_1}$.

Combined Mathematics 12 - II (Part B)

Answer five questions only.

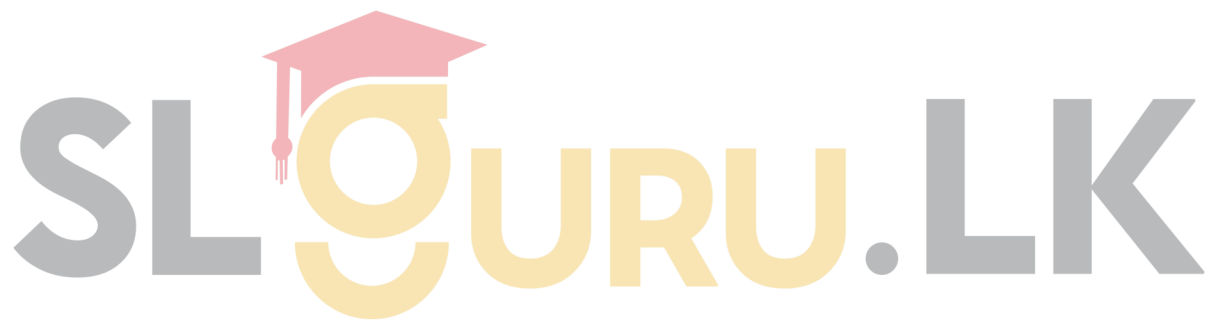
- 11) a) A train passes a station A at 40 kmh^{-1} and maintain this speed for 7 km and is then uniformly retarded, stopping at B , which is 8.5 km from A . A second train starts from A at the instant the first train passes A and being accelerated for part of the journey and uniformly retarded for the rest, stops at B at the same time as the first train.
- Find the total time for the journey.
 - What is the greatest speed obtained by the second train?
- b) A motor car X starts from rest at $t = 0$, moves with uniform acceleration. At $t = T$, another motor car Y starts at the same point with velocity u and moves with an uniform retardation of $2f$. If the motor cars meet each other, show that $2fT(u + fT) = u^2$.
- 12) a) Using the law of addition of vectors, show that the sum and difference of two vectors $\underline{a}$ and $\underline{b}$ gives by the diagonals of the parallelogram. If the sum of two unit vectors is an unit vector, show that the magnitude of their difference is $\sqrt{3}$.
- c) The position vectors of the points A, B and C relative to the origin O are $4\underline{i} + 2\underline{j}$, $\underline{i} + \underline{j}$ and $(k + 1)\underline{i} + 6\underline{j}$ respectively. Find the value of k ($k < 0$) such that $\hat{ABC} = 45^\circ$.
- c) In the parallelogram $OACB$, D and E are two points on BC and AC such that $BD:DC = 1:2$ and $AE:EC = 2:1$. F is the intersection point of OD and BE . The position vectors of A and B relative to O are $\underline{a}$ and $\underline{b}$ respectively. Show that $\overrightarrow{OF} = \frac{3}{10}(\underline{a} + 3\underline{b})$. If OB and CF intersect at P , find the ratio $OP:PB$.
- 13) a) OAB is an equilateral triangle of length of a side $2a$. C is the mid point of OA . The forces $4P, P$ and P act along the sides OB, BA and AO in the direction of the order of the letters respectively. Taking OA and OY (Parallel to CB) as X and Y axes respectively, express each force in the form $x\underline{i} + y\underline{j}$. Here $\underline{i}$ and $\underline{j}$ are unit vectors along OX, OY respectively. Show that the system of forces can be reduced to a single force $3P$.
- Also show that the above single force can be reduced to a couple of moment $2\sqrt{3}ap$ and to a like parallel force acting along the centre of the triangle.
- b) The points O, A, B and C are $(0,0)$, $(2,0)$, $(2,1)$, and $(0,1)$ respectively. The forces P, Q, R act along the sides OA, AB, BC respectively. If the resultant force of this system of forces lies on $x + 2y = 7$, find
- The resultant force in terms of P .
 - The moment of the couple such that the resultant lies on the line $x + 2y = 9$.

- 14) a) The position vectors of the points A and B relative to a fixed point O are $\underline{a}$ and $\underline{b}$ respectively. The points C and D lie on the OB and OA such that $OC:CB = 5:2$ and $OD:DA = 3:2$. The lines AC and BD intersect at E . Show that $\overrightarrow{OE} = \underline{b} + \lambda \left[\frac{3}{5} \underline{a} - \underline{b} \right]$. Here λ is a constant. Obtain a similar expression for $\overrightarrow{OE}$. Here find the position vector of the point E in terms of $\underline{a}$ and $\underline{b}$.
- b) The position vectors of the points A, B, C relative to a point O are $\underline{a}, \underline{b}$ and $\underline{c}$ respectively. The point P lies on the line BC and it is given that $\overrightarrow{PC} = \frac{1}{10} \overrightarrow{BC}$.
- (i) Find $\overrightarrow{OP}$ in terms of $\underline{b}$ and $\underline{c}$.
- (ii) If it is given that AP and BC are perpendicular to each other show that,
- $$(9\underline{c} + \underline{b}) \cdot (\underline{c} - \underline{b}) = 10\underline{a} \cdot (\underline{c} - \underline{b})$$
- c) Hence or otherwise prove that $(3\underline{c} - \underline{b}) \cdot (3\underline{c} + \underline{b}) = 0$, if OA, OB and OC are perpendicular to each other.
- 15) a) The mid points of the sides AB, BC, CD and DA of the rectangle are respectively P, Q, R and S respectively. Here $AB = 6a$ and $BC = 2\sqrt{3}a$. Six forces of magnitudes $15N, \lambda N, 5N, 10N, \mu N$ and $30\sqrt{3}N$ act along the sides PQ, QR, RS, SP, AD and CD respectively in the direction indicated by the order of the letters. Show that,
- (i) This system of forces cannot be in equilibrium.
- (ii) If this system of forces reduces to a couple, then $\lambda = -40$ and $\mu = 20$
- (iii) $\lambda = -40$ and $\mu = 30$, if the system reduces to a force of $10N$, acting along the direction AD .
- b) A string of length $2m$ is attached to two points A and B $1m$ apart in the same level. A smooth ring of weight $10N$ is suspended by the string and it is kept in equilibrium vertically below B , by applying a horizontal force P on the ring. Find the tension in the string and the magnitude of the force P .
- 16) a) In the rectangle $ABCD$, $AB = 4cm$ and $BC = 3m$. The forces of magnitudes $8, 7, 3, 2, 8, 7$ Newtons act along the sides $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CD}, \overrightarrow{DA}, \overrightarrow{AC}$ and $\overrightarrow{DB}$ respectively. Find the horizontal and vertical components of the resultant R . Hence find the distance from A , to the point which the line of action of the resultant cut AB .
Now a couple of forces of $9Nm$ is added to the system in the sense $ABCD$. Show that the line of action of the new resultant cut the side AB at a distance $2m$ from A .
- b) State the Lami's theorem for the equilibrium of three coplanar forces acting at a point. A string $ABCD$ attached to the two fixed points A and D on the same horizontal level supports two weights W_1 and W_2 at B and C respectively. In the equilibrium position, B is above C and the parts AB, BC and CD of the string inclined at acute angles α, β, γ to the vertical respectively.

Show that $\frac{W_1}{W_2} = \frac{\sin \gamma \sin (\beta - \alpha)}{\sin \alpha \sin (\beta + \gamma)}$ SLGURU.LK

- 17) a) A particle starts its motion with initial velocity u and moves with uniform acceleration a for a time t and obtains a final velocity v at a displacement of s . Using the velocity time graph for the motion of the particle, derive the equations of motion $v = u + at$, $S = \left(\frac{u+v}{2}\right)t$, $s = ut + \frac{1}{2}at^2$ and $v^2 = u^2 + 2as$.
- b) A particle is projected vertically upwards with a velocity $u \text{ ms}^{-1}$ and after t seconds another particle is projected vertically upwards from the same point with the same velocity. Prove that,
- (i) they meet after a time $\left(\frac{t}{2} + \frac{u}{g}\right)$ from the first projection.
- (ii) Particles meet at a height $\frac{4u^2 - g^2t^2}{8g}$.
- c) From a tap drops of water fall within equal time intervals. When one drop of water falls, earlier drop has travelled a distance of $\frac{1}{4} m$ when the distance between the two drops has increased to $\frac{3}{4} m$, find the distance travelled downwards by the first drop. ($g = 10 \text{ ms}^{-2}$)

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